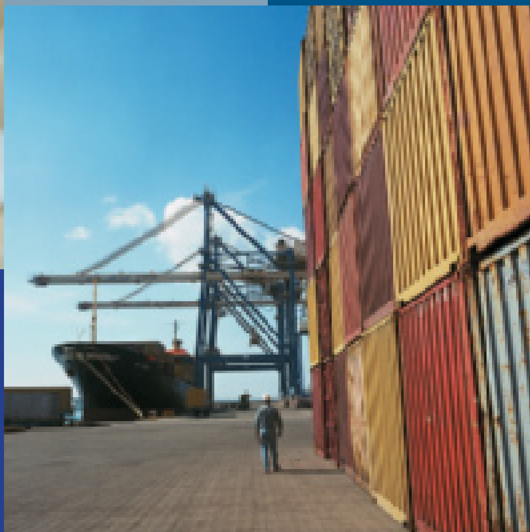
STUDENTS AS MATHE

If your students learn only one thing in your mathematics class, what would you want that to be? What do you want them to take with them as they walk out your classroom door?

As teachers, we must be able to answer these fundamental questions. Sometimes the answer goes no further than what skills students must master to be successful in their next mathematics class. Sometimes we need to look at the larger context and consider what students need to take with them for the rest of their lives. I try to convince my students that the most important skills that they can take from my classroom are thinking, reasoning, and problem solving. Regardless of their chosen career path and whether or not they think that they will ever need mathematics, students must have these three skills to be successful in life.

To teach students to think, reason, and solve problems, I had to make some changes in my approach to the curriculum. Too often in traditional mathematics classes, concepts are taught in distinct units. Regarding functions, first we teach everything there is to know about linear functions, and then we move on to quadratic functions. Too often, students are left with the mistaken assumption that there is no connection between the two topics. As a result, they do not get to practice recognizing the information in a practical setting, outside a particular unit in their textbook. Consequently, when they encounter a word problem, they assume that it fits the model of the particular function that they are currently studying. Students are not required, then, to be as creative or flexible in their mathematical thought as would be necessary in most real-life settings.

DEBT: ADVENTTR/ISTOCKPHOTO; REGISTER: FOTOFROG/ISTOCKPHOTO; FISH: TRISTAN LOW/ISTOCKPHOTO


Five problems—relating to gas mileage, the national debt, store sales, shipping costs, and fish population—require students to use functions to connect mathematics to the real world.

Jennifer L. Jensen

MATICS CONSULTANTS

If students are going to develop reasoning and thinking skills, use their mathematical knowledge, and recognize the relevance of mathematics in their lives, they need to experience mathematics in meaningful ways. Only then will their mathematical skills be transferrable to all other parts of their lives. According to NCTM's *Focus in High School Mathematics: Reasoning and Sense Making* (2009, pp. 9–10), reasoning habits that promote the development of flexible thinking include the ability to apply previous concepts to new problems, the ability to make connections across different contexts and different representations, and the ability to solve problems collaboratively. To promote such flexible mathematical thought, I have my algebra students become mathematics consultants.

THE OBJECTIVE

The goal of this activity is to have students learn how to apply mathematical knowledge in different situations and use this knowledge to make inferences and draw conclusions. I organize students into groups of three, and each group becomes a mathematics consulting firm. The choice of the title "mathematics consultant" is deliberate; it encourages students to interact with the data. Working collaboratively, students are presented with different scenarios and previously collected data. The scenarios represent a broad range of disciplines so that students might begin to see the relevance of mathematics in various situations. However, the real-life application of these scenarios is limited because the issues inherent in the scenarios depend on more factors than those investigated in the problems. The

value lies in students having the opportunity to choose the appropriate function to model the data.

The learning outcomes of the project include identifying different types of functions; analyzing real-world data; identifying dependent and independent variables; determining which type of function would most closely model the situation; using technology, including spreadsheets and graphing calculators, to create graphs and find best-fitting equations; using the equations to make predictions and inferences; and using technology to present conclusions to the class. Students are required to use multiple representations of functions, including verbal, graphical, algebraic, and tabular representations. These objectives align directly with those in the Algebra Standard for grades 9–12 set forth in NCTM's *Principles and Standards for School Mathematics* (2000, p. 296).

PROCEDURE

Before students form the mathematics consulting firms, I give them a short assessment to test their knowledge of functions, concentrating on linear, quadratic, exponential, and cubic functions. This simple tool asks them to work in two directions: to identify a graph by its equation and to graph a function given its equation. The same test is used as a posttest after students complete the project.

The first time I used this exercise only three of twenty-one students could name two or more types of functions. The only function that students could identify by sight or draw a picture of was a linear function. Only two students could name an application of functions. Not one student demonstrated mastery (over 80%) with regard to knowledge of functions. These results surprised me because we had studied different types of functions throughout the year. However, these results confirm my belief that students struggle to recognize mathematical concepts outside the unit in which these concepts are taught.

Then, as a class, we discuss the different types of functions—linear, quadratic, cubic, quartic, exponential, and logarithmic—that we have studied throughout the year. Although students do not always recall these functions at first, with prompting they can usually demonstrate some knowledge. After the discussion, I give them a sample data set that we analyze together, and we review how to use both the graphing calculator and a spreadsheet program, such as Excel®, to input the data and generate the best-fit equation.

After this thorough review, students meet in their groups and are given scenarios from five different "clients" who have hired their firm to analyze and interpret a specific set of data, draw conclusions, and make recommendations. Each scenario represents a different type of function. After students complete the different scenarios, each group chooses one situation to present to the class, using PowerPoint®, a format that they might one day use to make a presentation to a client.

A SYNOPSIS OF THE SCENARIOS

Scenario 1: Gas Mileage Legislation

A local congressman must decide whether or not to vote for a bill to reduce the speed limit, the argument being that gas mileage improves at lower speeds. Students are given a chart of the gas mileage of a particular car at different speeds (see **fig. 1a**). When they plot the data on the computer or calculator, they discover that the function that most closely models the data is a quadratic function (**fig. 1b** shows a sample graph generated by one group of students). Students must use the equation to determine the optimum speed for cars to travel, make a prediction about gas consumption at higher speeds, and make a recommendation to the congressman about how to vote on this legislation. The data for this scenario come from NCTM's Illuminations lesson "Paying for Your Wheels" (Marquez).

Scenario 2: National Debt

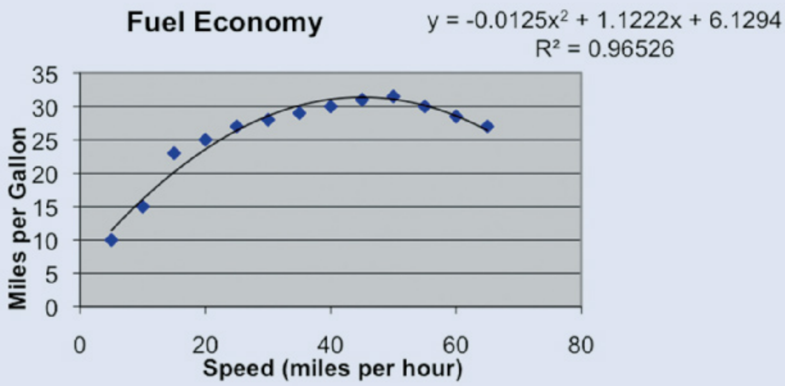
A U.S. senator seeks a mathematical analysis regarding the national debt before she can make a decision regarding a vote on an economic bailout package. Students must research the national debt for each decade, beginning in 1791, found on the Bureau of Public Debt's website (http://www.publicdebt.treas.gov/); see **table 1** for a sample of these data. When students enter the data in Excel, they determine that an exponential function most

Table 1 National Debt

Year	Amount (in millions of dollars)
1791	75
1801	83
1821	90
1841	5
1861	91
1881	2,069
1901	2,143
1921	23,977
1941	48,961
1961	28,8971
1981	997,855
2001	5,674,178

Speed	5	10	15	20	25	30	35	40	45	50	55	60	65
Mpg	10	15	23	25	27	28	29	30	31	31.5	30	28.5	27

(a)



(b)

Fig. 1 The data shown in (**a**) can be used to obtain a graph and an equation (**b**) that will help determine the speed that maximizes fuel economy.

closely models the data. Students must calculate the current national debt based on their equation and compare it with the actual national debt found on the website. Then students make inferences about the difference in these two numbers and make a recommendation to the senator regarding her vote.

Scenario 3: Clothing Store Sales

A clothing store manager wants help analyzing sales for the year so that he can make decisions regarding staffing and inventory. Using the total sales for each month, students create a spreadsheet and graph the data. See **figure 2a** for the data set provided to the students (Stewart, Redlin, and Watson 2004). A quartic function appears to be the best model for these data (see **fig. 2b**). However, some students find that a fifth- or sixth-degree polynomial provides a better fit. This discovery also allows for discussion of whether a higher-degree polynomial function makes logical sense in this scenario. After determining the function that most closely models the data, students draw conclusions about the store's profitability at different times of year. They also make recommendations to the store owner regarding staffing and inventory.

Scenario 4: Shipping Costs

The Civil Aeronautics Board wants to analyze shipping costs. Students are given data regarding the cost of shipping freight a certain distance. When they graph the data, they discover that a linear function most closely models this information. Once they determine the equation, they make predictions regarding the cost to ship crates varying distances. In addition, on the basis of the derived equation, students make recommendations to their client regarding the necessary budget for shipping.

Scenario 5: Fish Population Study

In the final scenario, students analyze data collected by the National Oceanic and Atmospheric Administration (NOAA) regarding the length and age of a certain species of fish (Stewart, Redlin, and Watson 2004). The details of this scenario are based on my own experience aboard a NOAA vessel conducting a study on Pacific hake. Some background on this study, the methods of data collection, and the data given to the students are provided in **figure 3**. After analyzing the data, students determine that the data are best modeled by a cubic function, and they make predictions regarding the age and length of the fish. Then they make recommendations to NOAA about their findings.

SOME FINDINGS

Students spend about seven days in the computer lab working with their group. As I monitor them during the project, they debate which function best models each scenario. As they deliberate and reconcile their differences, they exhibit flexible mathematical thinking. The groups are able to come to their decisions correctly without my prompting, although occasionally they ask me if they have drawn the right conclusion. The most questions

arise on the first day of the project, and these usually relate to how to use the calculator or computer. As the project progresses, however, students gain proficiency in both calculator and computer skills.

The first time I used this project, each group correctly identified the function that most closely modeled the given data. The groups also identified the dependent and independent variables correctly 80 percent of the time, although the graphs were correct 100 percent of the time. Making inferences and recommendations proved more difficult for students, but they were able to make accurate predictions based on the equations generated. After the first administration of the posttest, all twenty-one students were able to name the five functions that were presented in the different scenarios, with five students naming additional functions. Further, all twenty-one students could define the concept of a function and identify applications of functions, including some not in the scenarios.

At the end of the project, students complete a short survey. They often describe their experience as "fun," "useful," and "interesting." One student wrote, "It was interesting to learn about the different problems in the world." Another student indicated that he liked working "on stuff that is relevant." Overall, the students' responses to the project are positive, and they demonstrate improved knowledge of functions, their applications, and evidence of flexible mathematical thought.

Total Sales for 1 Year

Month	Sales (in dollars)
January	8,000
February	18,000
March	22,000
April	31,000
May	29,000
June	21,000
July	22,000
August	26,000
September	38,000
October	40,000
November	27,000
December	15,000

(a)

(b)

Fig. 2 Clothing sales based on the data shown in (**a**) can be modeled with a quartic equation (**b**).

PROJECT BENEFITS

Students must understand that when they encounter mathematics in the real world, there will be no flashing neon sign indicating the appropriate function. They need to be able to make connections among different graphical representations, equations, and real-world data. And they need to be able to think and reason, both critically and creatively.

This activity provides students the opportunity to develop all these skills. I plan to add to my repertoire of scenarios to include other types of functions or other fields of study. This practice will enrich the experience for students and further promote flexibility of mathematical thought through teaching the mathematical standards.

REFERENCES

Bureau of Public Debt, Department of the Treasury. http://www.publicdebt.treas.gov/.

National Council of Teachers of Mathematics (NCTM). 2000. *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

———. 2009. *Focus in High School Mathematics: Reasoning and Sense Making*. Reston, VA: NCTM.

Marquez, Elizabeth. "Paying for Your Wheels." NCTM Illuminations. http://illuminations.nctm.org/LessonDetail.aspx?id=L708.

Stewart, James, Lothar Redlin, and Saleem Watson. 2004. *College Algebra*. 4th ed. Belmont: Brooks/Cole-Thomson Learning.

JENNIFER L. JENSEN, jjensen@warren.k12.in.us, teaches mathematics at Warren Central High School in Indianapolis while she pursues a PhD in curriculum and instruction (mathematics education) at Indiana State University in Terre Haute. Her interests are promoting critical thinking and curriculum development.

SCENARIO 5: FISH POPULATION STUDY

Client 5: The National Oceanic and Atmospheric Administration (NOAA)

The National Oceanic and Atmospheric Administration (NOAA) is a government organization that studies the ocean and the atmosphere. Every three to five years, it commissions a population study of Pacific hake, a type of fish that lives off the coasts of Washington, Oregon, and California. The study attempts to determine the health of the population and set fishing limits. It is important that the oceans not be overfished.

Analyzing the health of the hake population includes documenting the size and age of the fish. Obviously, each hake in the ocean cannot be counted, so a sample is taken. For some fish in this sample, NOAA scientists remove an ear bone, the *otolith*. The otolith can help determine the age of the fish because each year's growth results in a new ring added to the bone; these rings are similar to growth rings in trees. After removing this bone, scientists record the length of the fish it came from. These data can then be used to draw conclusions about the age and size of a fish.

During its most recent hake study, NOAA recorded the data below. NOAA has hired your mathematics consulting firm to analyze the data. Your job is to determine what type of function might best model these data. Then you will need to use this model to answer some important questions for your client.

Age (yr.)	Length (in.)	Age (yr.)	Length (in.)
1	4.8	9	18.2
2	8.8	9	17.1
2	8.0	10	18.8
3	7.9	10	19.5
4	11.9	11	18.9
5	14.4	12	21.7
6	14.1	12	21.9
6	15.8	13	23.8
7	15.6	14	26.9
8	17.8	14	25.1

Instructions

1. Using Excel, input the data into two columns and create a scatter plot. Don't forget to title the graph and label the axes.

2. Looking at the graph, determine what kind of function appears to represent the data best. ____________________
____

Add the trend line to your graph. Don't forget to display the equation and the r^2 value on your chart. Are you sure that you have the best equation? Try a few different equations to make sure that you get the best fit.

3. What is the independent variable? __________

4. What is the dependent variable? __________

5. What is the equation? ______________________________

6. What is the value of r^2? _____________ What does this value tell you? ______________________________________
____

7. How well does this function fit the data? _____________
____

8. Now enter the data into a graphing calculator. Use the instruction sheet if necessary.

   What window should you use?

9. X-max: _____________ X-min: _____________
   Y-max: _____________ Y-min: _____________

10. Suppose that a fisherman catches a fish 20 in. long. How old is the fish? (Use a graphing calculator to help you answer this question.) _____________

11. Do you think that these fish will keep getting larger their entire life? Why or why not?
____

12. How big do you think the fish should be before they are caught to eat? Why?
____
____

13. How could this information help scientists make decisions?______________________________________
____

You may want to use information from the Internet to help you answer some of these questions.

Fig. 3 The author's experience aboard a NOAA vessel provides a personal, real-world backdrop for scenario 5.